Time-dependent mixing in stratified Kelvin-Helmholtz billows: Experimental observations

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[1] Quantitative time-dependent laboratory measurements are made of the irreversible mixing caused by the development, saturation and turbulent breakdown of Kelvin-Helmholtz (KH) billows in an initially stable two-layer stratified shear flow of miscible fluids. The measurements are obtained using a light attenuation technique. Irreversible mixing is found to be strongly dependent on the life-cycle of the flow, in particular the characteristics of large-scale, quasi-two-dimensional billow merging events, which stir the flow and then trigger mixing during turbulent transition. **Citation:** Patterson, M. D., C. P. Caulfield, J. N. McElwaine, and S. B. Dalziel (2006), Time-dependent mixing in stratified Kelvin-Helmholtz billows: Experimental observations, *Geophys. Res. Lett.*, *33*, L15608, doi:10.1029/2006GL026949.

[2] Stratified shear flows, i.e., flows where both the fluid density and velocity vary with height, are ubiquitous in geophysics. It is well-known [*Drazin and Reid*, 1981] that under certain circumstances, such flows are susceptible to a beautiful instability, where the initial strip of vorticity in the shear layer rolls up into a quasi-periodic array of elliptical vortices, commonly referred to as Kelvin-Helmholtz (KH) billows. The properties of stratified KH billows have been widely studied in the laboratory [*Thorpe*, 1987; *Fernando*, 1991] and numerically [*Peltier and Caulfield*, 2003]. Of particular interest are the characteristics of the irreversible mixing associated with billows, since for flows with sufficiently high Reynolds numbers $Re = Ud/\nu$ (where the fluid's kinematic viscosity is ν, and U and d are the characteristic velocity and length scales of the shear layer respectively) KH billows trigger turbulence. In the atmosphere, the break down of KH billows is thought to be a major cause of clear air turbulence (a significant, and still poorly understood aviation hazard), while in the ocean, shear induced mixing plays a major role in the overall circulation and heat budget [*Wunsch and Ferrari*, 2004]. In the context of larger scale models, it is essential to parameterize the mixing properties of a stratified shear layer, as characteristic time (of the order of minutes) and length scales (of the order of metres to tens of metres) of the billows are vastly too small to be resolved in all but the most idealized process models.

[3] Indeed, since the life cycle of an initially laminar stratified shear layer is strongly time-dependent [*Caulfield and Peltier*, 2000; *Staquet*, 2000; *Smyth et al.*, 2001], for an appropriate parametrization, it is necessary to consider the time-dependent nature of the mixing events within the flow. Using the fundamental concept of available potential energy [*Lorenz*, 1955]. *Winters et al.* [1995] constructed an algorithm which allowed the time-dependent separation of large-scale reversible "stirring" from the irreversible "mixing" of fluid elements. Using this approach in direct numerical simulations [*Peltier and Caulfield*, 2003], significant advances have been made in understanding shear-induced mixing. However, there undoubtedly remains some controversy as to the relative significance of mixing processes associated with "pre-" or "post-" turbulent stages of the flow [cf. *Smyth et al.*, 2001; *Peltier and Caulfield*, 2003].

[4] A major shortcoming of three-dimensional direct numerical simulation has been the resolution-driven need to impose flow periodicity in horizontal directions, and thus to consider typically only one or two billows. Furthermore, there are inevitably issues of concern with the small scale processes which lead to irreversible mixing being correctly captured numerically. Evidence from two-dimensional simulations of long streamwise extent [*Scinocca and Ford*, 2000] points to the possibility of different dynamics through the interaction of modes of slightly different wavelength, which clearly cannot happen in short computational domains with enforced periodicity. On the other hand, laboratory experiments, in particular using a "tilting tank" [*Thorpe*, 1968], impose much weaker modal streamwise quantization conditions.

[5] Therefore, we have conducted a sequence of experiments using the technique developed by Thorpe (see *Thorpe* [1987] for a detailed review), whereby a stable stratification of miscible fluids is created in a tank that can be tilted to an arbitrary (though typically small) angle θ. The stratification consists of fresh water of density ρ_0 and brine of density ρ_1, $\rho_1 > \rho_0$. Diffusive processes during experimental set-up led to the initial density profile being very close to an error function, i.e.,

$$\rho = \left(\frac{\rho_1 + \rho_0}{2}\right)\left[1 - \Delta \mathrm{erf}\left(\frac{2z}{\sqrt{\pi}h}\right)\right]; \; \Delta = \frac{(\rho_1 - \rho_0)}{(\rho_1 + \rho_0)}, \qquad (1)$$

where h is the density layer half-depth. Since salt was used to stratify the water, the Schmidt number is $\nu/D \sim O(10^3)$ (where D is the molecular diffusivity of salt in water). As the tank is tilted, gravity caused the development of an accelerating shear layer, with characteristic half-depth $d = Rh$, where the scale ratio R is given by

$$R = \frac{1 + \left(1 + \frac{\pi\nu t}{h^2}\right)^{1/2}}{2}, \qquad (2)$$

at time t after tilting [*Thorpe*, 1985]. Since the characteristic velocity parallel to the tank base of each layer was

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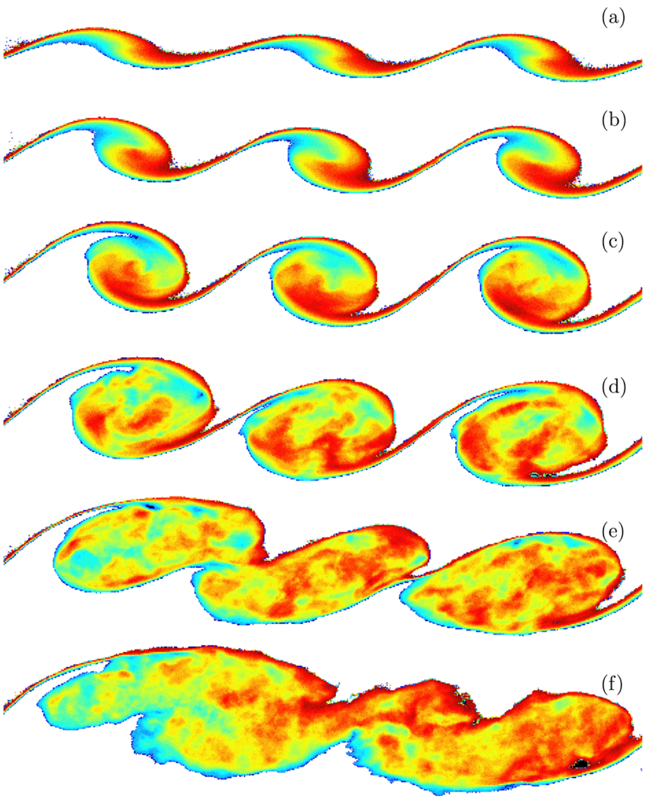

Figure 1. Contours of spanwise-averaged density for experiment A at nondimensional times $\tau = (t - t_{Akh})/t_{AT}$ =: (a) 7.5; (b) 10; (c) 12.5; (d) 15; (e) 17.5; (f) 20.

$U = t\Delta g \sin\theta$, the Reynolds number $Re = Ud/\nu$ and the bulk Richardson number $J = d\Delta g \cos\theta/U^2$ (where $g \sin\theta$ is the component of gravity parallel to the tank base) was simply

$$Re = \frac{\Delta Rhtg \sin\theta}{\nu}, \ J = \frac{R^2 h \cos\theta}{t^2 \Delta g \sin^2\theta}. \quad (3)$$

[6] The principal aim of this letter is to demonstrate the utility of experiments for the quantitative analysis of transitional KH billows, particularly in flows where the streamwise extent is longer than that which is typically considered in numerical simulation. Therefore, we focus on two experiments ("A" and "B") that exhibited significantly different behaviour in the streamwise direction. The experiments were conducted in a tank 300 × 15 × 7.5cm. There was a thin mesh on the lower surface of the tank which introduced a small-scale perturbation ($L \sim 2.2$cm) with length scale substantially smaller than the observed wavelength of the primary KH billows. Each experiment had three experimental inputs: the scaled density jump Δ, the tilt angle θ (kept constant, so the flow was accelerating) and the half-depth h of the initial density distribution. These parameters were $\Delta_A = 4.94 \times 10^{-3}$, $\Delta_B = 9.88 \times 10^{-3}$, $\theta_A = 10°$, $\theta_B = 6°$, $h_A = 0.510$cm and $h_B = 0.326$ cm. We intend to report on a more detailed parameter study in due course, but here we focus on showing how experiments can identify both quantitatively and qualitatively the time-dependent mixing characteristics of the flow.

[7] A critical time to determine was t_{kh}, the onset time of the primary instability. This time sets the initial values of the Reynolds number Re and Richardson number J. Conventionally, t_{kh} has been defined as the first instant at which the instabilities become visible, a qualitative method inevitably prone to subjective judgement. However, we identified t_{kh} for each experiment using a non-invasive optical technique, thus removing much of the subjectivity. The experiments were illuminated from behind and the flow evolution was recorded using a digital video camera. The images were post-processed using a light attenuation technique [*Hacker et al.*, 1996] that relates the amount of light that passes through the dyed fluid to the spanwise-averaged time-dependent concentration/density field. In particular, we tracked the midlevel of the shear layer (initially containing fluid of mean density $(\rho_1 + \rho_0)/2$ at all streamwise locations). Defining t_{kh} as the first time at which the power spectrum of the midlevel density had a discernible non-trivial peak (at the dominant wavelength of the primary instability), we found t_{Akh} = 6.14s and t_{Bkh} = 6.09s.

[8] At these times, the flow parameters for each experiment were $J_A = 0.123$, $J_B = 0.147$, $Re_A = 305$, and $Re_B = 268$. Compared to previous experiments, the onset Richardson number was somewhat large, though still sufficiently small to be consistent with the steady linear theory [*Howard*, 1961], while the onset Reynolds number was similarly somewhat smaller than previously reported values. These differences are largely due to the greater sensitivity of this method to the developing instabilities within the flow, although the presence of the mesh on the lower surface also played a role. After onset, the flow continued to accelerate (and so J decreased, while Re increased with time) and

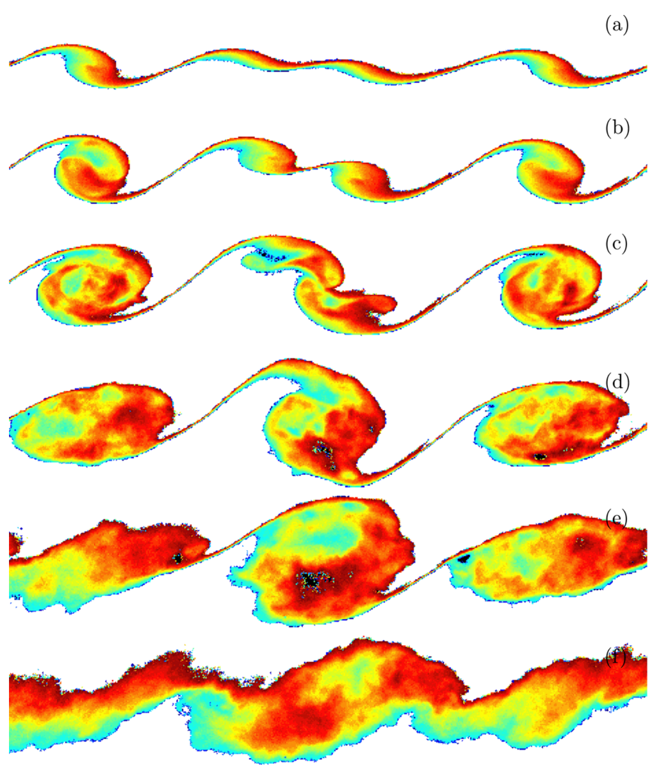

Figure 2. Contours of spanwise-averaged density for experiment B at nondimensional times $\tau = (t - t_{Bkh})/t_{BT}$ =: (a) 10; (b) 14; (c) 18; (d) 22; (e) 26; (f) 30.

the flow underwent a rapid transition to turbulence. The natural time scale of the flow was the turn over time scale $t_T = h/U_{kh}$, where U_{kh} was the flow velocity in each layer at the onset time t_{kh}. When analyzing the time-dependent flow evolution, we thus consider the non-dimensional time $\tau = (t - t_{kh})/t_T$.

[9] In Figures 1 and 2, we show the contours of spanwise-averaged density at various times in experiment A and experiment B respectively in the middle region of the flow domain. In this region, end effects were insignificant during the times shown, and the density distribution was close to periodic at the (shown) streamwise boundaries. Since $J_B > J_A$ and $Re_B < Re_A$, the evolution of the primary instability, as well as the subsequent flow transition, was more rapid in experiment A. There was also a clear difference in the large-scale behaviour of the two experiments.

[10] In experiment A, three vortices, with essentially the same wavelength, developed to finite amplitude, and then merged in a largely symmetric manner, leading to what was clearly a vigorous overturning of the flow. This strong overturning appears to have been possible because each billow grew substantially, thus extracting a significant amount of energy from the background flow. The behaviour of experiment B was markedly different, due to the combined effects of (uncontrolled) variation in the initial perturbation and (controlled) variation in the imposed flow parameters Δ, θ, and h. In the region of the flow shown, four billows developed, although from analysis of the spectrum, there was actually a competition between perturbations corresponding to three and five billows. Since $h_B < h_A$, it is unsurprising that smaller wavelength billows developed in experiment B. The evolution of these two competing perturbations was strongly time-dependent. Near the centre of the region shown, the billows that grew out of the smaller wavelength perturbation underwent rapid merger. This merger was on a smaller scale than in experiment A, and affected the larger scale billows strongly, in particular stopping the development of a large-scale merger overturning late in the experiment. Instead, each billow broke down without substantial interaction with its neighbours. This suppression of large-scale merger is not entirely surprising, since the bulk Richardson number is also slightly larger for this experiment, and it is possible that the enhanced stratification plays a role.

[11] Quantitative time-dependent measures of the mixing within the flow may also be obtained from the experimental data, following a modification of the approach of *Winters et al.* [1995]. In the tilted tank, the total potential energy (density) of the fluid may be defined as

$$E_p = \frac{g\cos\theta}{V}\int_V \rho z \mathrm{d}V \equiv \frac{g\cos\theta}{2L_z}\int_{-L_z}^{L_z} \langle\rho\rangle_{xy} z \mathrm{d}z, \tag{4}$$

where ρ is the density of the fluid, V is the volume of the region under consideration, $-L_z < z < L_z$ is perpendicular to the tank base, and $\langle . \rangle_{xy}$ denotes horizontal averaging.

[12] Formally, the potential energy (density) can be subdivided into two parts: the background potential energy E_b, defined as the minimum potential energy attainable through an adiabatic redistribution of ρ within the flow volume, and the available potential energy (density) $E_a = E_p - E_b$ which is available to drive motions within the flow. Essentially, this redistribution organizes the density distribution so that there are no horizontal density gradients, and the density decreases monotonically vertically, and so

$$E_b = \frac{g\cos\theta}{2L_z}\int_{-L_z}^{L_z} \rho(z^\star) z^\star(x,y,z,t)\mathrm{d}z^\star, \tag{5}$$

where $z^\star$ is the notional reference location of the sorted "background" density fluid element actually at (x, y, z, t). We choose to define potential energies with effective gravity given by $g \cos \theta$ and the vertical direction perpendicular to the tank base as this gives the most natural comparison with numerical simulations. It is important to appreciate that, if the experimental apparatus is left in its tilted state, surfaces of constant density would eventually be arranged at an angle θ to the tank base. We always terminate our experiment sufficiently early so that this final adjustment has a negligible effect on the flow's behaviour.

[13] Mixing, which in this context corresponds to irreversible changes in the density of fluid elements, directly modifies E_b but not E_a through irreversibly changing the reference location of some of the fluid density elements, and so the time-dependent evolution of E_b quantifies the extent of mixing within the flow [*Peltier and Caulfield*, 2003]. Indeed, the evidence from numerical simulations suggests that a substantial amount of mixing occurs when the total potential energy is actually decreasing, as the associated large increase in both kinetic energy and small-scale structure is highly conducive to mixing.

[14] Using the dye attenuation technique outlined above, it is possible to calculate a conservative approximation to the background potential energy. Experimentally, our technique enables us to measure the spanwise-averaged density $\langle\rho\rangle_y$, which in general is a function of x, z, and t. Averaging this density in the streamwise direction over a periodic region enables the calculation of the total potential energy E_P. From our experimental images, this density distribution can be rearranged adiabatically so that there are no streamwise variations in density, and as before, the rearranged density decreases monotonically with height. This rearranged density distribution can of course be used to define a spanwise-averaged background potential energy density $E_{by} \geq E_b$:

$$E_{by} = \frac{g\cos\theta}{2L_z}\int_{-L_z}^{L_z} \langle\rho\rangle_y \left(z_y^\star\right) z_y^\star(x,z,t)\mathrm{d}z_y^\star, \tag{6}$$

where, analogously, $z^\star{}_y$ is the notional reference location of the spanwise-averaged element at (x, z, t). Provided spanwise gradients are not large compared with streamwise gradients (as undoubtedly is the case during the development and initial breakdown of KH billows) it is reasonable to suppose that E_{by} is a useful approximation to E_b. The time evolution of E_{by} thus can show the processes by which mixing takes place.

[15] Therefore, in Figure 3, we plot the time evolution of the total potential energy density, the spanwise-averaged background potential energy density, and the remaining "available" potential energy density for each experiment. On each panel, we mark with vertical grid lines the times at which the density contours in Figures 1 and 2 are plotted.

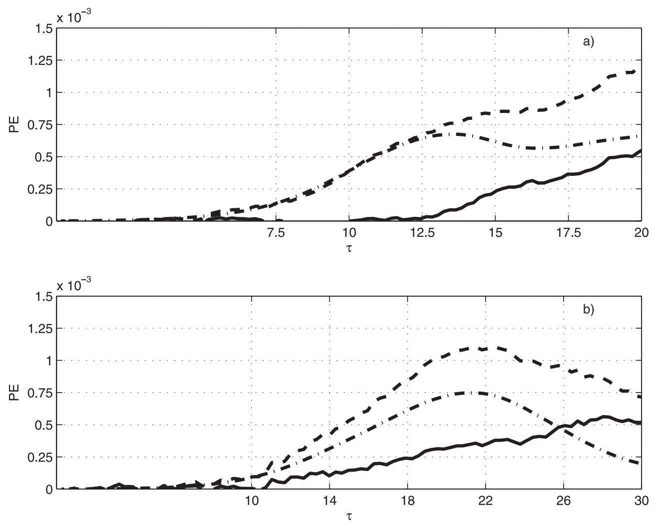

Figure 3. Plots of total potential energy E_P/K_{kh} (dashed line), spanwise-averaged background potential energy E_{by}/K_{kh} (solid line) and available potential energy $E_a/K_{kh} = (E_p - E_{by})/K_{kh}$ (dash-dotted line) against nondimensional time τ for: (a) experiment A; (b) experiment B.

The potential energy densities are scaled with the characteristic kinetic energy density of the flow at onset $K_{kh} = \overline{\rho}U_{kh}^2/2$, as this allows sensible comparison of the two experiments.

[16] In experiment A, the development and saturation of the three primary billows involved virtually no mixing, and both the available and the total potential energy increased markedly. Even regions of static instability did not correspond to mixing at this stage, as the density distribution could be sorted to a very close approximation to its initial profile. This is strong evidence that "pre-turbulent" phases of flow evolution do not play a dominant direct role in mixing, at variance with some numerical investigations [*Smyth et al.*, 2001]. This variance is very interesting, though caution must be exercised in drawing strong conclusions at this stage. Possible reasons for the variance may be the relatively short, and strictly periodic, numerical domain, differences in the character of the initialization of perturbations, or the acceleration of the flow in the laboratory. The relatively closer upper and lower boundaries or the somewhat higher overall stratification in the laboratory may also be important. There is a clear need for further work to investigate more fully this issue, but such a detailed parameter study is beyond the scope of this letter, where the principal focus is the demonstration of the utility of this new experimental approach to shear-driven mixing problems of this kind.

[17] Only when the billows reached their saturated amplitude did measurable mixing occur (at nondimensional time $t/t_{AT} \sim 13$), as available potential energy drove small scale turbulent motions conducive to mixing around the periphery of the primary billow cores. The primary instabilities should be thought of as catalytic for the development of mixing, as they "stored" potential energy that was then available to drive mixing as the instabilities undergo turbulent transition.

[18] The total potential energy continued to increase, albeit at a slower rate, because of the large-scale merging of the three primary billows and the associated onset of turbulence throughout the flow regime. This large-scale merging, due fundamentally to the primary billows reaching a large amplitude, led to the mixing continuing at approximately the same rate. Small scales, which initially were associated with statically unstable regions within the primary billow cores, were subsequently associated with the billow-billow merging process. The evidence from the evolution of the background potential energy strongly suggests that the specific mechanism for the development of secondary instability, onset of transition and the proliferation of small scales is not as important for mixing as the amount of energy which is available to drive those small scales, i.e., the "intensity" or "strength" of the turbulence transition itself.

[19] In experiment B, the behaviour was qualitatively different. The total potential energy reached a maximum value at $\tau \sim 20$, associated with the maximum amplitude of the smaller merged billows. However, this merger was actually counter-productive to the total amount of mixing which occurs within the flow, because it reduced (at least to some extent) the stored potential energy that was available for mixing due to the growth and saturation of the primary billows, and hence the intensity of the ensuing transition. This effect was exacerbated by the onset of mixing somewhat earlier in the flow evolution, due to overturning in the smaller billows as they merged. Indeed, the mixing rate within experiment B was little affected by the large-scale dynamics, illustrating that consideration of the large-scale structures within a flow can be very misleading as a guide toward actual mixing. In particular, even as the total potential energy of the flow decreased (due to the observable collapse of the two small merged billows), and hence the buoyancy flux was converting potential energy into kinetic energy, irreversible mixing increased the background potential energy at (to leading order) the same rate.

[20] We have demonstrated that it is possible to make quantitative time-dependent measurements of an estimate for the irreversible mixing caused by the development, saturation and turbulent breakdown of KH billows in an accelerating laboratory stratified shear layer. A particular attraction of using the technique discussed here is that it allows consideration of longer flow regions with less restrictive boundary conditions than are conventionally considered numerically. This allows the effects of large-scale merging events (as illustrated by the behaviour of experiment A) and interaction between primary instabilities with different wavelengths (as illustrated by experiment B) to be analysed quantitatively. Such studies are complementary to numerical simulations, and promise to lead to new insights and improvements in mixing parameterizations for use in larger scale models. Having demonstrated in this letter the feasibility of using this dye attenuation technique for mixing measurements in the laboratory, it is now appropriate (and of interest) to conduct a systematic parameter study of the mixing dynamics of such shear-driven unstable flows.

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