Air flow and snow transport through arboreal wind breaks

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Introduction

Artificial wind breaks of negligible thickness have been studied in the field, in wind tunnels and numerically and their properties are well understood qualitatively, namely that low porosity wind breaks provide the greatest shelter immediately behind a wind break but due to recirculation and increased turbulence the shelter distance is reduced. Quantative results are also available from experiments and numerical simulations though there is as yet little convergence in predictions.

Experiments with natural shelter belts are harder to perform and harder to analyze. Apart from the difficulties in finding suitable experimental sites, deciding which aerodynamic parameters are necessary and sufficient to describe a shelter belt is unclear. Their finite width means that pressure and velocity fields can vary considerably within the shelter belt so that simple parameterizations of, for example, height, width and porosity are unlikely to be effective. Porosity (as defined in terms of relative velocity reduction) is in fact likely to be a strongly varying function of velocity.

The study of the effects of wind breaks on snow transport processes has progressed in recent years and current simulations of snow fences provide qualitative agreement with field and wind tunnel experiments. However the effects of natural shelter belts on snow transport processes has received almost no study to date, and all the important questions remain unanswered.

Objectives

Wind breaks and natural shelter belts are designed for many purposes including controlling cornices and avalanches, improving visibility, controlling snow drifting, preventing

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land erosion, and protecting crops from high winds. The last three of these being of major agricultural significance. The extent to which different aims can be achieved is poorly understood as research has concentrated on single objectives. However, by combining a model for wind flow through natural shelter belts with a model for snow transport processes an integrated theory is possible that will be capable of predicting wind sheltering effects and snow drifting.

For finite length shelter belts and when the wind direction is not normal to a shelter belt a three dimensional model is necessary. However, for angles close to normal and for shelter belts much longer than their width the system can be approximated by averaging along the length of the shelter belt resulting in a two dimensional model.

Basic equations of modeling

The basic are equations are those for a two phase flow coupling the air, treated as an incompressible Newtonian gas, and the snow particles approximated by a volumetric density field and a velocity field. Inter-particle interactions in the flow are neglected. The region is averaged over the length of the shelter belt so thus the shelt belt only appears in the form of an additional drag force on the air and particles and no longer constitutes a (very complicated) boudnary condition.

The subscripts $_a$, $_s$ and $_b$ designate air, snow or shelter belt properties respectively. Thus c_a, c_s and c_b are the volumetric concentrations of air, snow and shelter belt material. ν is the air viscosity, d the particle diameter, $\mathbf{u}_a$ the air velocity field and $\mathbf{u}_s$ the snow particle velocity field.

There are three coupling drag forces between the snow and the air, the snow and the barrier and the air and the barrier. The standard form of the drag equation for an individual particle is

$$D = \rho A \mathbf{u}|\mathbf{u}|C_D, \tag{1}$$

where ρ is the density of the fluid, $\mathbf{u}$ the velocity of the fluid A the cross sectional area of the object and C_D the dimensionless drag function which varies with the objects Reynolds number $R_p = d|\mathbf{u}|/\nu$. d being the object characteristic length and ν the fluids viscosity.

For the air-snow interaction the effective density is reduced by the volumetric concentration of the air $\rho = \rho_a c_a$ and $A \propto c_s/d$. Thus

$$D_{as} = \rho_a c_a c_s/d\mathbf{u}_{as}|\mathbf{u}_{as}|C_{Das}, \tag{2}$$

where $\mathbf{u}_{as} = \mathbf{u}_s - \mathbf{u}_a$. There is a wide choice of drag functions available. One choice that has proved successful for snow particles is

$$C_{Das} = 18/R + 3R^{-2/3}. \tag{3}$$

For the air shelter belt interaction the drag force can be written

$$D_{ab} = \rho_a c_a A \mathbf{u}_a|\mathbf{u}_a|C_{Dab}, \tag{4}$$

where $A(\mathbf{x})$ is the leaf area density and C_{Dab} the drag coefficient for unit leaf area density, approximated as a constant.

The snow shelter belt interaction also takes a similar form. If the snow shelter belt collisions have a coefficient of restitution e and the normal collision rate is assumed proportional to leaf area density then

$$D_{sb} = \rho_s c_s A(1+\mathrm{e})\mathbf{u}_s|\mathbf{u}_s|. \tag{5}$$

In the full set of equation below we define $\mathbf{F}_{as} = \mathbf{u}_{as}|\mathbf{u}_{as}|C_{Das}/d$, $\mathbf{F}_{ab} = A\mathbf{u}_a|\mathbf{u}_a|C_{Dab}$ and $\mathbf{F}_{sb} = A(1+\mathrm{e})\mathbf{u}_s|\mathbf{u}_s|$. Conservation of air

$$(c_a)_{,t} + \nabla\cdot(c_a\mathbf{u}_a) = 0, \tag{6}$$

conservation of snow

$$(c_s)_{,t} + \nabla\cdot(c_s\mathbf{u}_s) = 0 \tag{7}$$

and conservation of volume

$$c_s + c_a + c_b = 1. \tag{8}$$

Conservation of air momentum (The Navier Stokes equation), after dividing by $c_a\rho_a$,

$$\mathbf{u}_{a,t} + (\mathbf{u}_a\cdot\nabla)\mathbf{u}_a = -1/\rho_a\nabla p + 1/c_a(\nabla\nu\cdot\nabla)(c_a\mathbf{u}_a) + \mathbf{g} + c_a\mathbf{F}_{as} + \mathbf{F}_{ab} \tag{9}$$

and conservation of snow momentum (divided by $c_s\rho_s$)

$$\mathbf{u}_{s,t} + (\mathbf{u}_s\cdot\nabla)\mathbf{u}_s = -1/\rho_s\nabla p + \mathbf{g} - \rho_a c_a/\rho_s\mathbf{F}_{as} - \mathbf{F}_{ab}. \tag{10}$$

Snow particle interactions are not accounted for thus there is no particle stress term.

The air boundary conditions are no slip $\mathbf{u}_a = \mathbf{0}$ on the ground. The air and snow boundary conditions with the shelter belt are approximated for by the drag forces. Particle ground boundary conditions cannot be directly included since erosion and deposition fluxes depend non-linearly on the individual particle velocities and the air speed, thus cannot be accurately reflected by an average particle velocity field. One approach would be to include higher order particle velocity fields and then develop boundary conditions using a splash function.

However, since we only wish to solve these equations for average quantities it is much simpler to apply phenomenological particle flux boundary conditions after Reynolds averaging. In snow transport processes the volumetric density of the snow is low so we can approximate the equations by neglecting all except leading order terms in c_s. If we also assume that the volumetric concentration of the shelter belt is small then variations in c_a and c_b can also be neglected. The gravitational force is removed from the air momentum equation by adding $\rho_a\mathbf{x}\cdot\mathbf{g}$ to the pressure. The $\mathbf{g}$ in the snow momentum equation is then replaced by $\mathbf{g'} = (1-\rho_a/\rho_s)\mathbf{g}$.

Reynolds averaging using the Boussinesq law and looking for steady state solution the equations become

$$\nabla\cdot\mathbf{u}_a = 0, \tag{11}$$

$$\nabla \cdot (c_s \mathbf{u}_s) = \nabla \cdot (\nu_t/\sigma_c \nabla c_s), \tag{12}$$

$$(\mathbf{u}_a \cdot \nabla)\mathbf{u}_a = -1/\rho_a \nabla p + (\nabla \nu_t) \cdot \mathbf{u}_a - 2/3\nabla k + c_s \mathbf{F}_{as} + \mathbf{F}_{ab} \tag{13}$$

$$(\mathbf{u}_s \cdot \nabla)\mathbf{u}_s = -1/\rho_s \nabla p + (\nabla \nu_t) \cdot \mathbf{u}_s - 2/(3c_s)\nabla(c_s k_S) - {}'\mathbf{g} - \rho_a/\rho_s \mathbf{F}_{as} - \mathbf{F}_{ab}. \tag{14}$$

Turbulent energies of the particle and gaseous phase are linked by $k/k_s = 1 + t_p/t_l$, $t_l = 0.41k/\epsilon$ and

$$1/t_p = |\mathbf{u}_{as}| C_{Das} \rho_a/(d\rho_s) \tag{15}$$

For addition closure equations we use turbulent energy conservation

$$\mathbf{u}_a \cdot \nabla k = \nabla \cdot (\nu_t/\sigma_c \nabla k) - \sigma \nabla \mathbf{u}_a - \epsilon + S_k \tag{16}$$

and turbulent energy dissipation

$$\mathbf{u}_a \cdot \nabla \epsilon = \nabla \cdot (\nu_t/\sigma_\epsilon \nabla \epsilon) - C_{1\epsilon}\epsilon/k(\sigma\nabla) \cdot \mathbf{u}_a - C_{2\epsilon}\epsilon^2/k + S_\epsilon \tag{17}$$

where the Reynolds stress tensor

$$\sigma = 2/3k - C_\mu k^2/\epsilon[\nabla \mathbf{u} + (\nabla \mathbf{u})^T] \tag{18}$$

and

$$S_k = -2c_s k/t^*[1 - \exp(1 - t^*\epsilon/2k)] \tag{19}$$

and $S_\epsilon = -2\epsilon/t^* c_s$ and $t^* = d^2\rho_s/(18\rho_a\nu)$ the turbulent viscosity is related to the turbulent kinetic energy and dissipation by

$$\nu_t = C_\mu k^2/\epsilon \tag{20}$$

The constants are chosen with the following values

$$C\mu = 0.09 \; \sigma_k = 1; \sigma_\epsilon = 1.22 \; \sigma_c = 0.5 \; C_{2\epsilon} = 1.92 \tag{21}$$

0.1 Boundary conditions

The interface with the ground is a turbulent boundary layer so

$$\mathbf{u}_s = \mathbf{u}_*/K \log(\mathbf{n}.\mathbf{x}/h_0), \tag{22}$$

Where $\mathbf{n}$ is a surface normal K suVon Kármán's constant, h_0 the roughness height and $\mathbf{u}_*$ the friction velocity. These are connected with other turbulence properties near the ground by.

$$kK^2 = \sigma_\epsilon(C_{\epsilon 2} - C_{\epsilon 1})|\mathbf{u}_*|^2 \tag{23}$$

and

$$\epsilon = k|\mathbf{u}_*|/(\mathbf{n}.\mathbf{x}). \tag{24}$$

For the particle flux conditions we assume that the rate of entrainment is proportional to the excess shear stress over some limiting friction velocity $\mathbf{u}_{*t}$ below which no entrainment occurs and that there is some limiting concentration c_{max}. That is

$$\nabla(c_s\mathbf{u}_s) = \rho_a\psi(1 - c_s^2/c_{\mathrm{max}}^2)(\mathbf{u}_*^2 - \mathbf{u}_{*t}^2) \text{ for } |\mathbf{u}_*| \geq |\mathbf{u}_{*t}|. \tag{25}$$

This is equivalent to assuming that the actual friction velocity decrease according to $\mathbf{u}_*^2 - (\mathbf{u}_*^2 - \mathbf{u}_{*t}^2)c_s^2/c_{\mathrm{max}}^2$. ψ is a constant depending on the snow properties. For deposition we postulate a deposition coefficient χ and the equation

$$\nabla(c_s\mathbf{u}_s) = -c_s\chi(1 - c_s^2/c_{\mathrm{max}}^2)(\mathbf{u}_{*t}^2 - \mathbf{u}_*^2) \text{ for } |\mathbf{u}_*| < |\mathbf{u}_{*t}|. \tag{26}$$

Expected results

These equations are a closed system of elliptic nonlinear partial differential equations and can be solved numerically by a variety of methods. For the simple geometry involved (a half plane) straightforward grid methods should be applicable. Solution of these equations will then give average wind speeds and turbulent energies as well as the snow deposition/entrainment rate as a function of position. To the extent that the ground profile is unchanged by snow transport this will enable the evaluation of the effectiveness of different shelter break designs. Since visibility is related to snow particle concentration this is also available.

If the shape of the snow drifts is important the parabolic time dependent equations must be solved using an adaptive finite element grid, which will be much more computationally intensive.

The large number of parameters in the problem also poses difficulties. The various turbulence parameters cannot be calculated theoretically and come from experimental studies. The entrainment, deposition and maximum concentration coefficients are available from wind tunnel data and discrete element method simulations of saltation. The leaf area density and leaf area drag coefficients cannot be directly measured. These functions can only be calculated by comparing the results of the simulation with field data. Once this has been done for a variety of shelter breaks it may then be possible to develop an empirical relation between measurable quantities and these functions.